GEOMETRY AND ART

Optical Illusions and the Fibonacci Sequence

Big Ideas

Unit of Instruction

This unit emphasizes the connection between mathematics and optical illusions that can be created using the Fibonacci sequence.

Geometry Concept

- Learn to recognize and use the Fibonacci sequence and its applications

Rationale

This introductory project will help get your artistic students “hooked” on geometry. After examining many uses of optical illusion in architecture, mathematics, and visual art, students will gain a greater understanding of how the Fibonacci sequence can be utilized to create their own optical illusions.

NCTM 9-12 Standards

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply transformations and use symmetry to analyze mathematical situations.
- Build new mathematical knowledge through problem solving.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Recognize and use connections among mathematical ideas.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Idaho Content Standards

- G.4.1.1 Analyze properties and determine attributes of two-and three-dimensional objects.
- G.4.1.2 Explore congruence and similarity among classes of two-dimensional objects and solve problems involving them.
- G.4.1.3 Establish the validity of geometric conjecture using inductive and deductive reasoning.
- G.4.4.1 Draw and construct representations of two-dimensional geometric objects using a variety of tools.
- 9-12.VA.3.1.1 Select and apply media, techniques, and processes effectively and with artistic intentions.
- 9-12.VA.3.1.4 Select and visualize visual, special, and temporal concepts to enhance meaning in artwork.

Background

Vocabulary

Contrast	Optical Illusion
Diagonal Line	Reflection
Dynamic	Sequence
Fibonacci sequence	Square Plane
Horizontal Line	Symmetry
Intersection	Vertical Line
Op Art	

Math Instruction (pre- or post-project)

Introduce Leonardo Pisano Fibonacci
Ruler measurement skills
Geometric shapes: rectangle, triangle, parallelogram, pentagon, hexagon, etc.
Golden Section

Driving Question

Project Objective

To gain an understanding of the Fibonacci sequence, the students will be introduced to optical illusions.

Questions to be Answered

What is Op Art?
What are optical Illusions?
How can optical illusions be classified?
Where do you notice patterns in nature?
Who was Leonardo Fibonacci?
Why is Fibonacci important?
What is a Fibonacci sequence?
Why is a Fibonacci sequence important?
How can a Fibonacci sequence be used to create an optical illusion?

Materials

Materials Required

- 8.5 X 11 inch paper
- Drawing paper
- Black markers of various size tips
- Rulers
- Pencils
- Erasers (acrylic or kneadable)

References

http://www.cs.brown.edu/stc/outrea/greenhouse/nursery/optical_illusions/home.html#Activitytwo
http://www.walterwick.com/opticaltricks_goingup_bts.htm
http://archrecord.construction.com/news/daily/archives/060328chicago.asp
http://mathworld.wolfram.com/FibonacciNumber.html
http://users.skynet.be/J.Beever/fountain.html

Seckel, Al. *Incredible Visual Illusions.* Arcturus, 2006.
Seckel, Al. *SuperVisions: Geometric Optical Illusions*. 2005

Lesson Outline

Description of Activity

- Overview of Op Art, using the internet
- Fibonacci Checkerboard Art
- Work Day

Day One: Introduction

Establish Set

- Teacher led discussion about patterns that students have seen in nature (zebra stripes, butterfly wings, sunflower, pinecone, shells, star fish, etc.)
- Ask students to help identify the pattern types. 5 minutes

Introduction to Optical Illusions

- The teacher will show a variety of optical illusions, and identify them as optical illusions, to get the students' attention. See attachment. 5 minutes

Definitions of Optical Illusions

- Split the students into four or five groups. For each group, pass out a folder of resource materials on optical illusions.
- Ask each group to come up with a definition of optical illusion and five examples of what they collectively think are illusions.
- In twenty minutes they select a spokesperson from the group and that person makes their presentation.
- Each group presents and the class votes on the best definition and uses that definition for the remainder of the lesson.
- All the above suggested topics get covered with this approach except the students initiate the interest by sighting their examples. 15 minutes
- Split students into groups of five.
- Distribute a variety of samples of the seven basic types of illusions to each group (page 12). Allow students time to discuss and evaluate what they are seeing.
- Have each group make a presentation about the classifications of their samples. 20 minutes

Day Two: Fibonacci Checkerboard Art

5 minutes: Fibonacci Teacher Presentation

- Presentation on Leonardo Fibonacci and the Fibonacci sequential numbers (see page 6).

Checkerboard Divisions

1. A checkerboard is the division of a square plane into equal square units (see figure 1, page 8).

2. If those horizontal and vertical units are not equal, but instead are units of sequential Fibonacci numbers, interesting designs result. This is a Fibonacci checkerboard (see figure 2, page 8).

3. Sequential Fibonacci units can be used **symmetrically** on a side (see figure 3, page 7).

4. Whether the Fibonacci units are connected vertically, horizontally, or diagonally, the resulting design can still be colored (or shaded) in an alternating checkerboard fashion (see figure 4, page 8).

5. Fibonacci Checkerboard Art can be applied to other geometric shapes such as triangles, parallelograms, pentagons, etc. The resulting designs are characterized by a certain dynamic energy and, often, optical illusions as well (see figure a-c, page 9). 10 minutes

Guided Practice

- The teacher, using the overhead, and the students, using pencil and paper, will go through a step-by-step process on how to create a simple square Fibonacci checkerboard. 15 minutes

Assignment and Design

- Assign the students to create an optical illusion based on a Fibonacci sequence.
- Caution students to use careful measurements with a ruler.
- Students will need to work on rough designs so they will be ready to work on a finished project.
- Assign and review requirements for the reflection paper. 15 minutes

Day Three: Student Work Time

- Answer any student questions from the group.
- Working time with the teacher answering questions and offering suggestions. 40 minutes.

Teacher Note: Assess if students need one more day in class to work with teacher assistance, or assign as homework. Assign due date a few days later so students may invest extra time in their projects.

Assessment

Rubric

Optical Illusion (100 points possible)
Creativity: 20 points Students may get an optical illusion or line design.
Neatness: 20 points Accuracy in measurement is paramount.
Accuracy: 60 points Students will use Fibonacci sequence in their project
Reflection (20 points possible)
Reflection paper may address two mathematical and two artistic discoveries.
Technical writing: 5 points Content: 15 points

Background Material for Optical Illusions

PROOFS

Many optical illusions “appear” in a certain way and are hypothesized to indicate a specific concept or truth but can then be proven false.

So it is with proofs in geometry. One may make certain conjectures or hypotheses that may or may not be proven false.

FIBONACCI

Leonardo of Pisa (c1175-1250) was also known as Leonardo Pisano, Leonardo Bonacci, Leonardo Fibonacci, or just Fibonacci. He was an Italian mathematician and is labeled “the most talented mathematician in the middle ages.” His father, Bonaccio, directed a trading post east of Algiers in North Africa. This is where he learned about the Hindu-Arabic number system.

Fibonacci is most known for spreading the Hindu-Arabic numeral system, rather than the Roman numeral system, through his “Book of Calculations,” the “Liber Abaci.” He applied the Hindu-Arabic numeral system to bookkeeping and the practical importance of the new numeral system. Using just the numerals 0-9, one could write any number. His book promoted a much easier addition and subtraction system. It also taught lattice multiplication, fractions, and decimals (although use of decimal numerals did not become widespread until the invention of the printing press almost three centuries later.

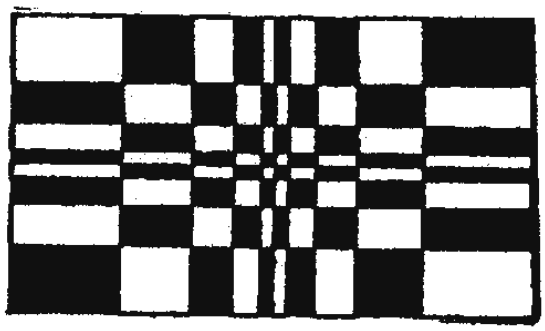
Fibonacci’s book proved the Pythagorean Theorem, using the Hindu-Arabic number system, and it became the standard text for teaching. His book also proposed, and solved, a problem involving the growth of a hypothetical population of rabbits based on idealized assumptions. The solution, generation by generation, was a sequence known as the Fibonacci Sequence (1, 1, 2, 3, 5, 8, 13, . . .)

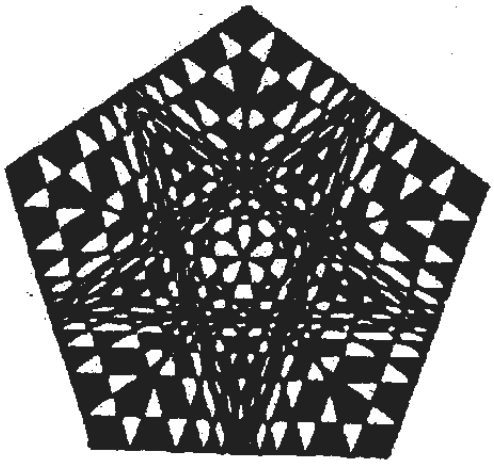
Optical illusions may be created in many ways. One way is to use the Fibonacci sequence to create Fibonacci checkerboard art.

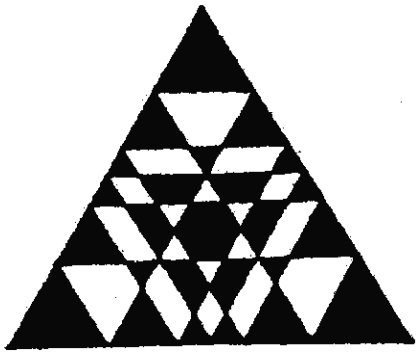
Ideas for Further Independent Student Project

Quick Tips in Design: www.atpm.com/10.02/design.shtml
The Fibonacci Spiral is a mirrored duplication rotated around the center to create the pattern found in sunflowers.
Research on artists or art that use optical illusions: M.C. Escher, Victor Vasarely, Andy Goldsworthy, Bridget Riley, Hajime Juchi, Edna Andrade, Islamic tile designs, and Celtic knots.
Possible Senior Project: Create an original Op Art project inspired by artist, such as Escher; create a sidewalk chalk art project inspired by Julian Beever; create an optical illusion room with distorted perspectives; create a Celtic knot made out of a variety of mediums; and complete an architectural plan with floor tiles set in the Fibonacci sequence.

Fibonacci Checkerboard Art can be applied to other geometric shapes such as triangles, parallelograms, pentagons, etc.
The resulting designs are characterized by a certain dynamic energy and, often, optical illusions as well.







For more incredible information on Fibonacci Numbers, check outhttp://www.fibonacci.net/. Near the bottom, click on the related link "The Fibonacci Numbers and the Golden Section." You will then be accessing one of the world's truly great web sites. It originates in England and contains more information on Fibonacci Numbers and related topics than you could possible ever want to know! Good Luck!

These materials are taken from:

- *FASCINATING RUN Fascinating Activities with Intriguing Numbers* by Trudi Hammel Garland
- *Fibonacci Checkerboard Art* (A Poster) by Trudi Hammel Garland & Nancy Wells Brewer

Additional Recommended Resources:

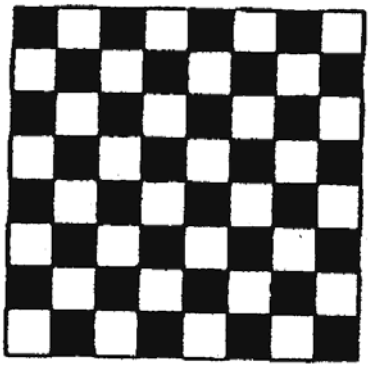
- *FASCINATING FIBONACCIS Mystery and Magic in Numbers* by Trudi Hammel Garland (Poster)

- *THE GOLDEN SECTION* by Garth Runyon (Poster)

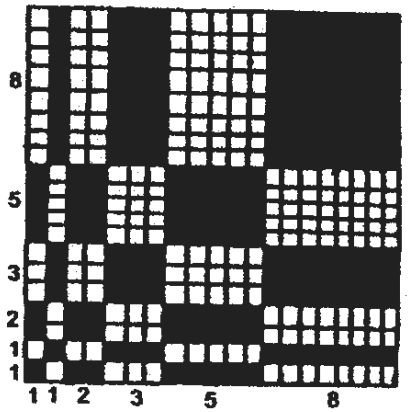
Dale Seymour Publications 1-800-321-3106

Fibonacci Checkerboard Art A Poster
Trudi Hammel Garland & Nancy Wells Brewer

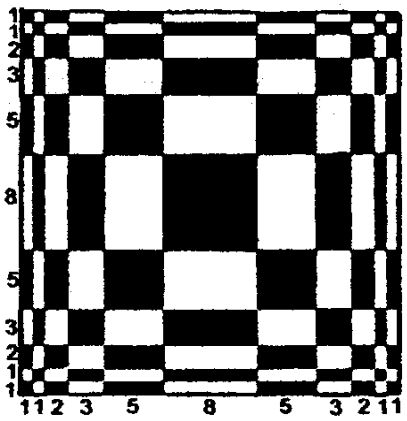
1. A checkerboard is the division of a square plane into equal, square units achieved by the intersection of horizontal units and vertical units of equal dimension.



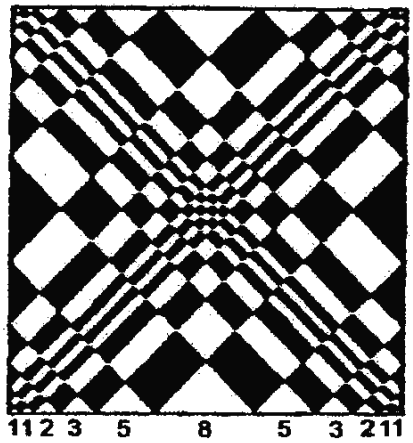
2. If those horizontal and vertical units are not equal, but instead are units of sequential Fibonacci numbers, interesting designs result. This is a Fibonacci Checkerboard.



3. Sequential Fibonacci units can be used symmetrically on a side.



4. Whether the Fibonacci units are connected vertically, horizontally, or diagonally, the resulting design can still be colored (or shaded) in an alternating checkerboard fashion.

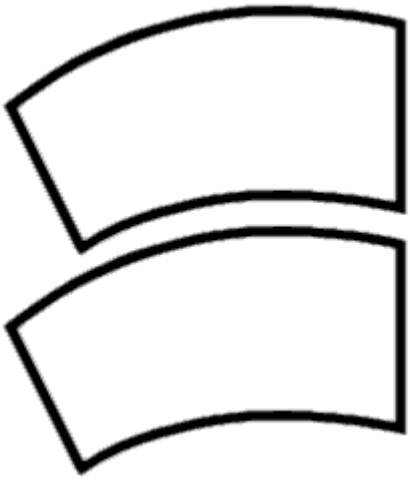


CoolOpticalIllusions.com

How illusion has been used through the ages in realism and abstraction:
from the invention of perspective in the 14 hundreds to twentieth century artists such as Albers and Escher.

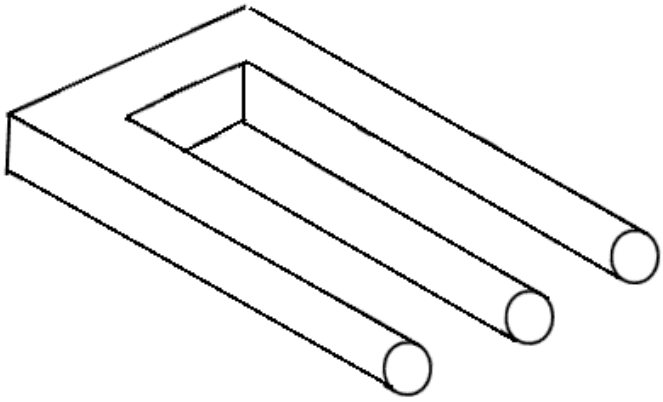
How illusion is used in cultural designs:
Advertising, textile, fashion, and interior design etc

Arch Illusions



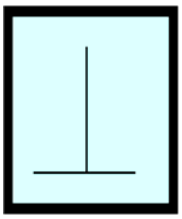
Here's a simple magic trick/ optical illusion you can make. Just take a piece of paper and fold it in half, then cut a rainbow shape like you see above. Your arch/rainbow shape can be even skinnier than this picture. As you hold them together, it'll look like the one that is above the other one is much smaller, when in fact they are the exact same size!
Some optical illusions are tricks-they at first appear to be drawings of real objects, but actually they are impossible to make, except on paper.

Tuning Fork?

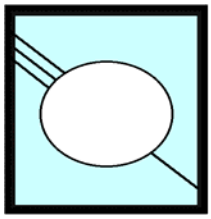


What kind of Illusion?

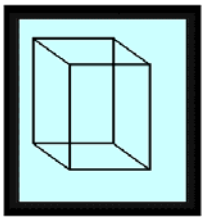
http://www.cs.brown.edu/stc/outrea/greenhouse/nursery/optical_illusions/home.htm



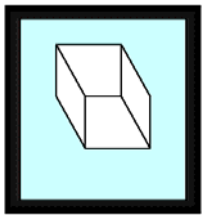
Height Width



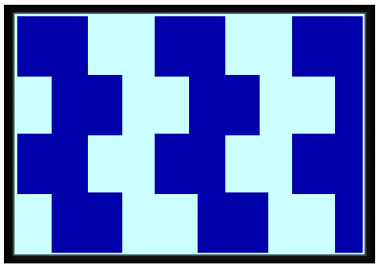
Interrupted Extent



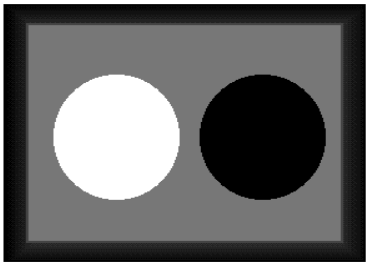
Equivocal



Illusion of Depth



Illusion of contrast



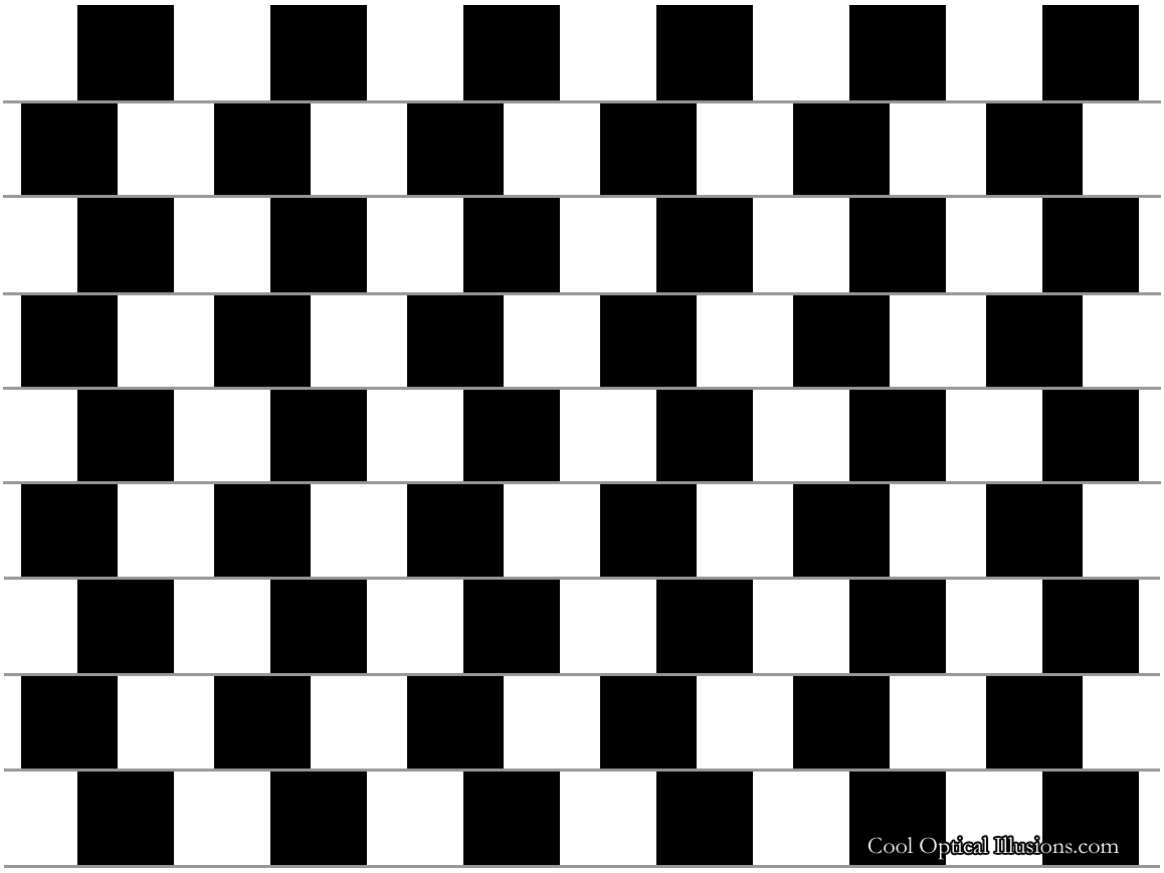
Illusion of contour

The Seven Basic Kinds of Illusions

1. Height-Width Illusion- an upright line segment appears longer nearby equal horizontal line segments.
2. **Illusion of Interrupted Extent**- the distance between the portions of an interrupted line or figure appears to shrink.
3. **Illusion of Contour**- an open or empty figure appears to have greater area or capacity than an equal closed or filled figure.
4. **Equivocal Illusion**- the drawing of a three dimensional object appears to have two or more interpretations.
5. **Illusion of Depth**- line or figures are distorted as the result of a point, line, or figure appearing to be behind or in front of another.
6. **Illusion of Contrast**- surrounding lines of figures cause other lines or figures to appear to have distorted dimensions or shapes.
7. **Non-Classified Illusions**- an optical illusion that does not fall into one or more of the above categories.

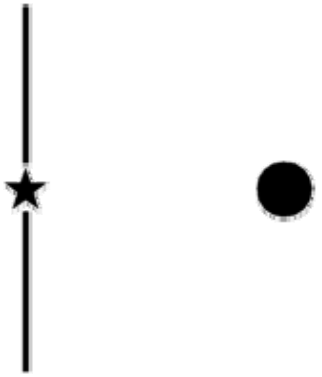
Straight Lines?

http://www.walterwick.com/opticaltricks_goingup_bts.htm

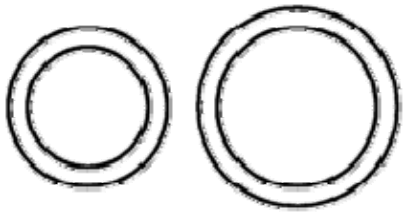


Some Optical Illusions www.jimloy.com/puzz/illusion.htm

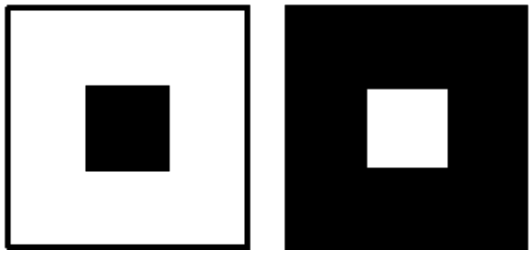
Here are some geometric optical illusions:



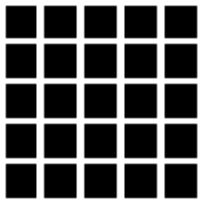
1. Blind spot: Your eye has a blind spot, where the optic nerve is attached to the eye. Close your right eye and stare at the black dot. Move forward or backward (depending on the size of your computer screen), and at a certain distance, the star will appear to disappear, and the vertical line will appear continuous. Your brain fills in the missing data for this blind spot.



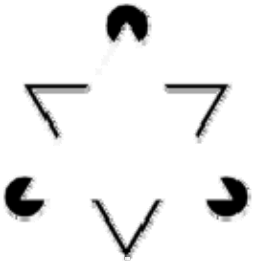
2. Delboenf's illusion: Here we have four circles. The outer circle on the left, and the inner circle on the right are of the same size, but the right one appears larger.



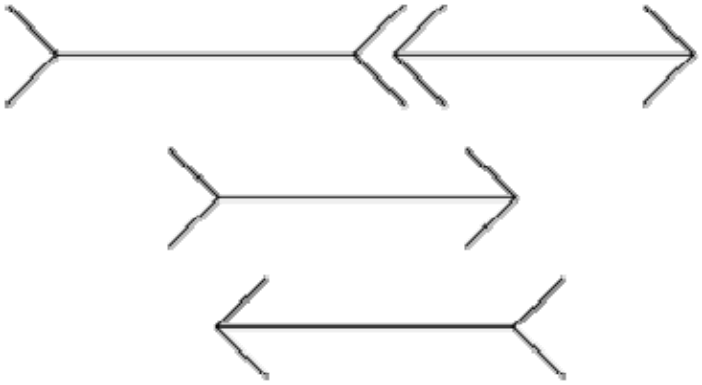
3. Helmholtz' irradiation illusion: The two figures are of identical size, but the small white square (inside the larger black square) looks larger than the small black square.



4. Hermann's grid illusion: Looking at this grid produces the illusion that there are gray spots or squares at the intersections of the streets or canals.

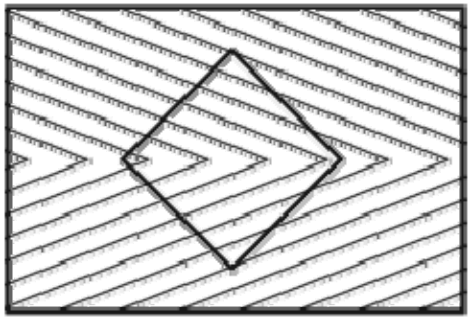


5. Kanizsa's triangle: This produces the illusion of a nonexistent white triangle. Sometimes the black angles are not shown, just the black dots with slices taken out. And it is possible to bend the sides of the nonexistent triangle.

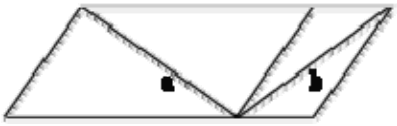


6. Müller-Lyer illusion: The horizontal line segment on the far left looks longer than the other one, even though both segments are of the same length (within the limits of my paint program).

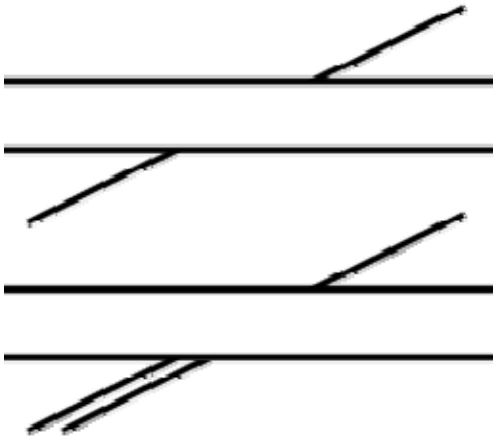
Below that is a different illusion. The bottom horizontal line segment appears to be to the right of the other horizontal segment.



7. Orbison's illusion: The outer rectangle (really) and the square appear distorted. A circle would also appear distorted.

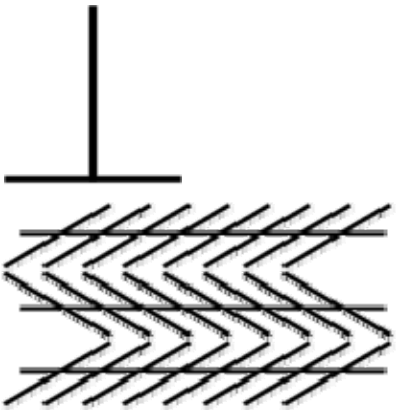


8. The parallelogram illusion: Diagonals a and b are of equal length.



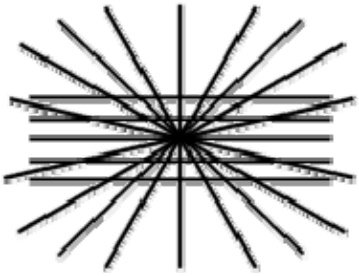
9. Poggendorff's illusion: In the top diagram, the diagonal lines do not appear to line up, but they are the same line. Below that we have the same illusion as it is often shown, with a second segment which seems to line up with the other line on the other side.

10. Ponzo's illusion: The upper horizontal line segment appears to be longer.



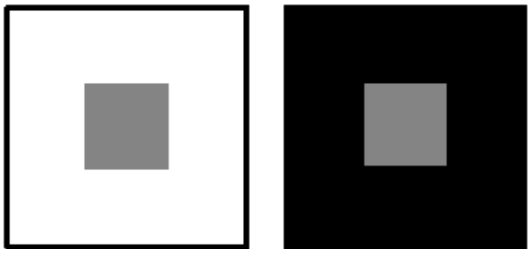
11. The vertical-horizontal illusion: The vertical line appears longer than the horizontal line, but they are of equal length. Sometimes this illusion is done as a top hat.

12. Zollner's illusion: The horizontal lines are parallel.

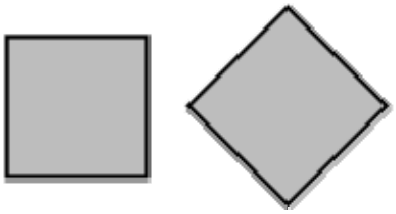


13. Another illusion: The horizontal lines are straight and parallel.

14. A famous illusion: The left-most object appears longer.

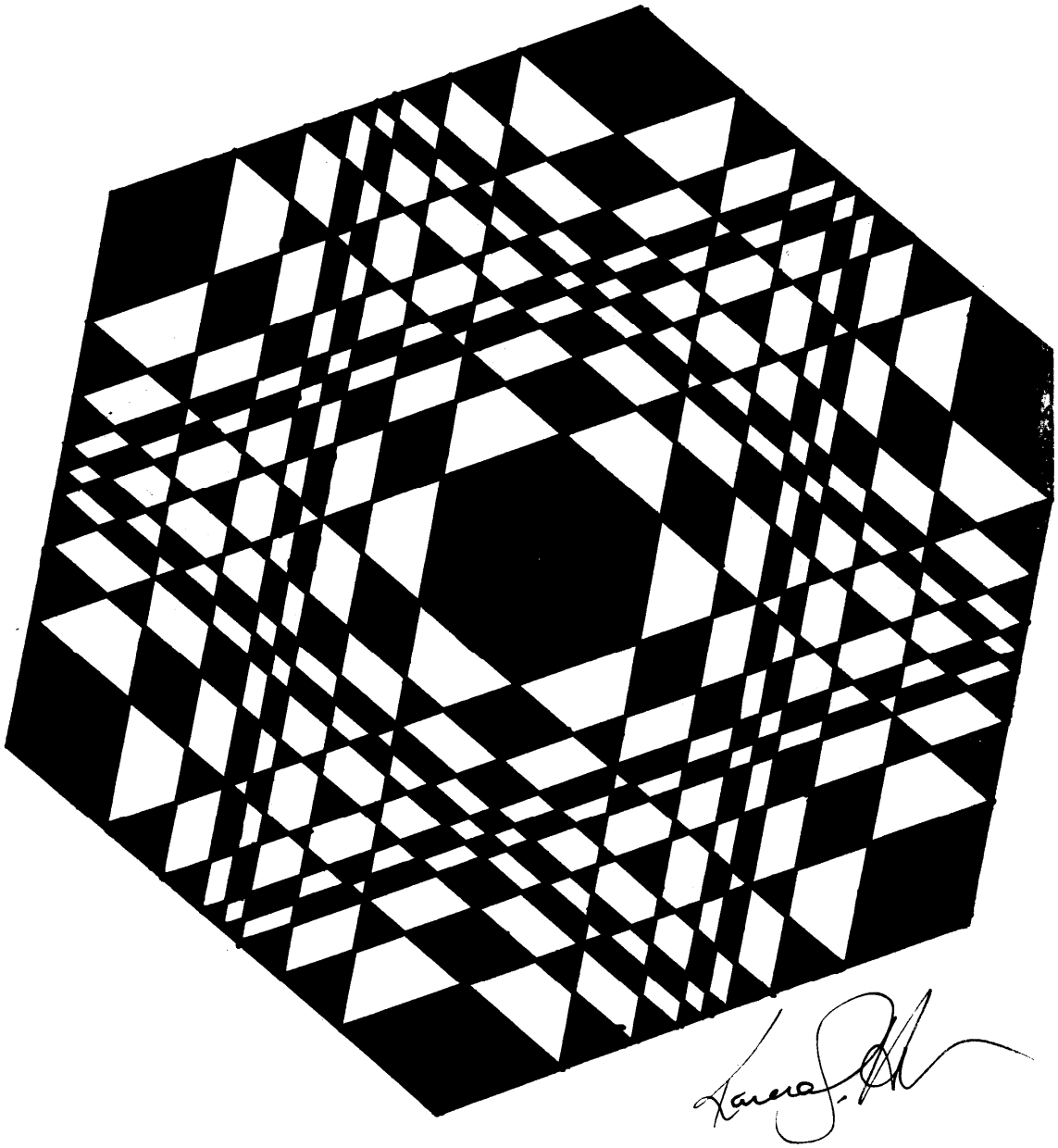


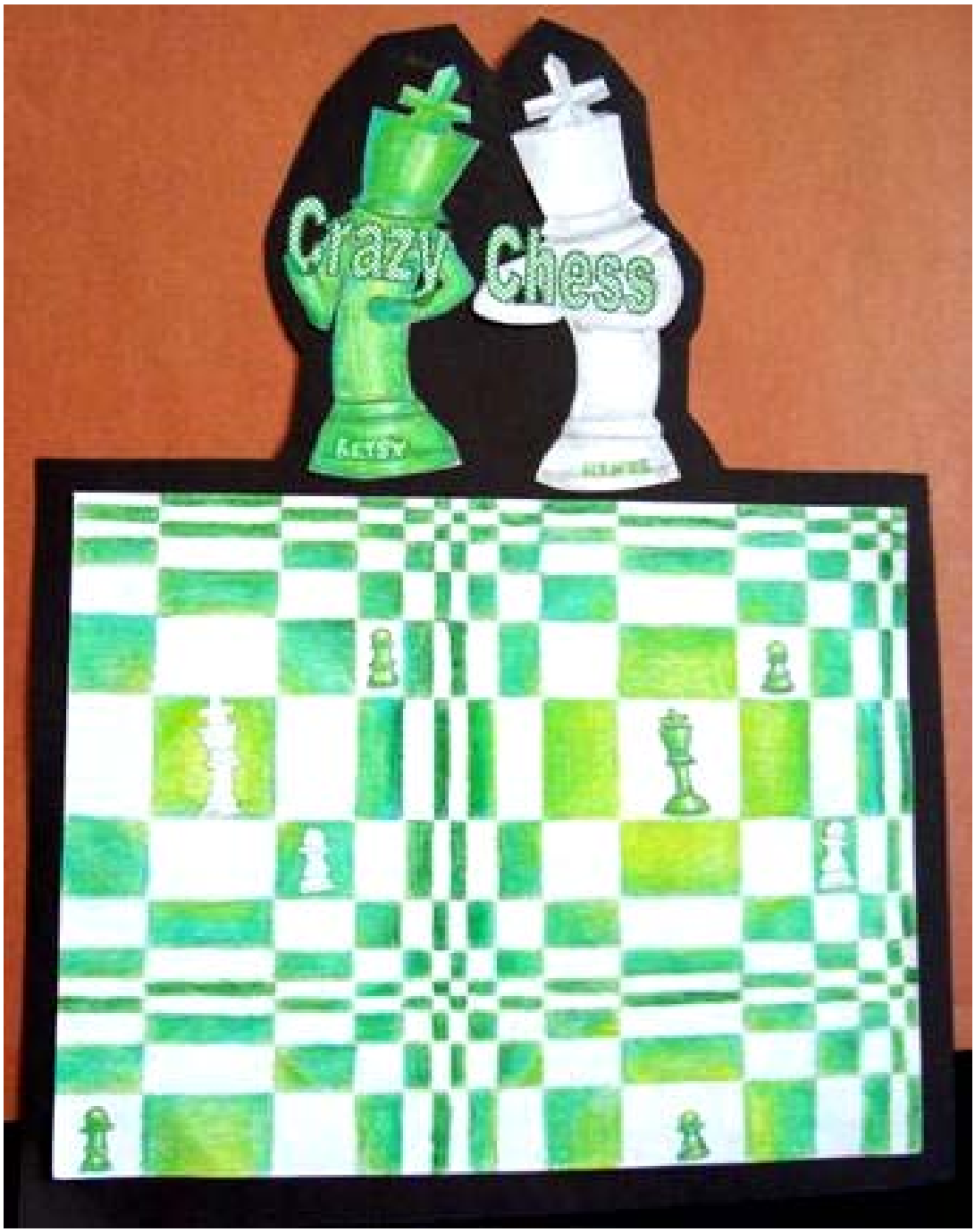
15. Shades of gray: The left-most gray square appears darker (and smaller as in #3, above). They are the same shade of gray. See Checker Shadow for a more dramatic version of this.



16. Two squares: Sometimes the right-most square looks larger.

Examples of Fibonacci checkerboard art




Crazy Chess